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Effective Young's moduli of quasilayered bars using bar resonance frequencies

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Rectangular-bar specimens with nonhomogeneous (quasilayered) porosity distribution were prepared from an iron powder. Using these specimens, differences were observed in effective Young's modulus values determined experimentally by means of specimen flexural vibrations with bending in the plane of quasilayers and in the plane perpendicular to quasilayers. To account for these differences, general expressions for resonance frequencies of flexural vibrations of model quasilayered bars were derived in a theoretical way. The model quasilayered bar means a prismatic bar of rectangular cross section, with material Young's modulus varying continuously only along one of transverse directions. Substituting theoretical resonance frequencies, instead of experimentally measured ones, into the routine evaluation procedure of the dynamical resonant method, theoretical values of effective Young's modulus were calculated. In majority of theoretically investigated simple model examples with lowered modulus of surface regions the effective modulus determined by means of the frequency of sample flexural vibration perpendicular to layers is lower than the effective modulus determined by means of the vibration parallel to the layers. These theoretical results explain qualitatively the effective modulus differences obtained for our quasilayered samples in an experimental way.

1. Introduction

Powder metallurgy (PM) structural parts produced by pressing and sintering are used to a considerable extent in automotive engines, transmissions etc. The most distinctive feature of these parts is their inherent porosity. Since it is impossible to fully consolidate most metal powders at ambient temperature, reasonable effort and costs, the residual porosity has to be accepted. Numerous publications have dealt with the relationships total porosity-mechanical properties. In particular the Young's modulus, which is a property not linked to plastic deformation or to singular defects in the material, has been a target for investigations (e.g. [1–4]).

Generally, it is agreed upon that the mechanical properties of PM ferrous components are adversely affected by the porosity. This is particularly true for fatigue loaded parts. Recently, attempts have been made to produce gears for automotive transmissions by PM techniques which is however successful only if sufficiently low porosity can be obtained at least at the surfaces subjected to contact fatigue loading. A promising approach is selective densification at the loaded surface, e.g. by rolling [5]. This results in a quasilayered structure with fairly dense surface regions and considerably more porous core. On the other hand, also the reverse structure, with porous (e.g. self-lubricating) surfaces and strong solid cores might be useful.

For structural parts, besides the total porosity, also the layered structure (i.e., intentional or accidental nonhomogeneous porosity distribution) should have a pronounced effect on the overall properties of sintered components. Within this work it is shown how the measurement of the effective Young's modulus by means of the dynamical resonant method is affected by variation in porosity along the specimen cross section.

Strictly speaking, the modulus determined by means of measurements of the frequencies of flexural vibrations is the effective flexural modulus of the particular bar being tested. For uniform, homogeneous and isotropic bar the flexural modulus is same as the Young's modulus of bar material. So, throughout this article, we call this flexural modulus as an effective Young's modulus E^{v}.

Within the Classical Lamination Theory (e.g. [6]) it was established that for actual layered specimens (with sharp planar interfaces between constant-parameter

TABLE I Chemical composition of the starting iron powder

C (%)	Si (%)	Mn (%)	P (%)	S (%)	O_2 (%)	N_2 (%)	Rest insoluble in HCl (%)	Fe
0.02	0.03	0.079	0.003	0.008	0.124	0.006	0.117	Rest

layers) the effective flexural moduli are different when the bending plane is parallel and perpendicular to the layers. In this case the effective flexural moduli E^{v} are different from the effective extensional or tensile moduli E^{t}, too [6].

One of aims of this article is to show in a theoretical way that also quasilayered structure, with more or less continuously varying porosity (and consequently effective material Young's modulus value) along one direction of the bar cross section, can really lead to the observed different values of effective moduli determined by means of the resonant frequencies of flexural vibrations perpendicular and parallel to quasilayers.

2. Specimen preparation and testing

Specimens with quasilayered structure were prepared from an iron powder by pressing and sintering with subsequent hammer forging.

The experimental programme was carried out on the basis of a water atomized iron powder WPL-200, produced by means of Mannesmann equipment at ZVL-METALSINT a.s. Dolný Kubín, Slovakia. The chemical composition of the powder is shown in Table I. The particles of the powder are of approximately equiaxed type with the characteristic articulation of the surface.

To obtain the specimens with low and nonhomogeneously distributed porosity, the following technological procedure was used: Samples were compacted at 600 MPa to four different heights (12, 10, 8, and 7 mm). The compacts were then sintered for 2 h at 1120 °C in a retort silit furnace, the atmosphere being cracked ammonia (75%H_2 + 25%N_2). The dew point of the atmosphere was −20 °C. The sintered test bars were densified by hammer forging at 1100 °C in hydrogen atmosphere to final height of 6 mm. The volume density of the resulting rectangular bars was determined by measuring the dimensions and by weighing. The porosity obtained was in the range from 2.4 to 7.6%. The final size of the bars being tested were 6 mm × 6 mm × 90 mm. The frequency of specimens natural vibrations was measured employing the apparatus Gringo Sonic MKS "Industrial" at University of Vienna. The frequency of flexural fundamental mode for the bar with free ends was used for the modulus evaluating.

3. Experimental results

Metallographic study revealed that the technological procedure used for preparing the specimens led to nonhomogeneous porosity distribution within the samples. The porosity is varying along the pressing (and forging) direction and its value decreases from the surface to the bulk of specimens (Fig. 1). So the samples can be assumed as "quasilayered", with layers perpendicular to the pressing (forging) direction. The term "quasilayered" is used as, firstly, small porosity fluctuations are

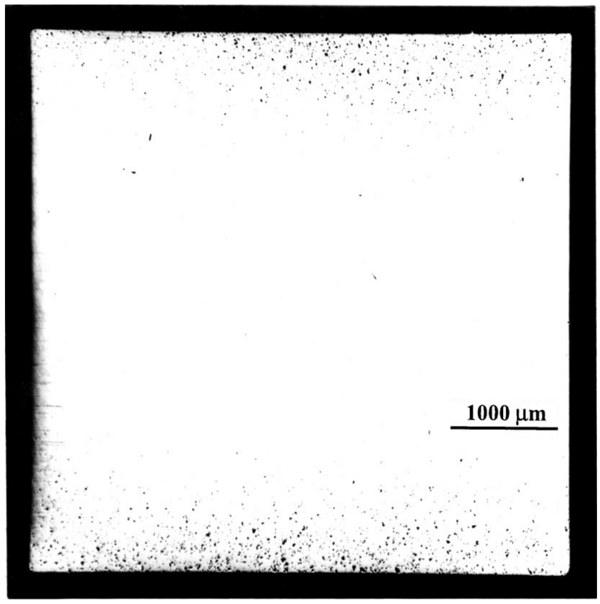

Figure 1 Polished cross section of a sample of total porosity 2.7%, made from a sintered powder iron. In this figure, pressing direction is the vertical one.

observed also in the direction perpendicular to pressing and, secondly, transitions between various porosity regions along the pressing direction are continuous rather than sharp, stepwise.

The dynamical resonant method [7] was used for determining the Young's modulus. The frequency of fundamental flexural mode of the prismatic bar of rectangular cross section was employed. The experimental equipment configuration was such that vibrations of the bar with free ends were realized. In this case the modulus is evaluated by means of the expression [7].

$$E^{\mathrm{v}} = 0.94642 \frac{\rho L^4 f^2}{t^2} T$$
$$T = 1 + O\left(\left(\frac{t}{L}\right)^2\right) \tag{1}$$

Here ρ is density of material, L represents the length of specimen, t is the specimen cross-sectional dimension in the direction of vibration and f represents the measured resonance frequency of the fundamental flexural mode. T is a correction factor allowing for the finite value of the ratio t/L. Symbol $O(x)$ represents the quantities of order less or equal to the order of quantity x.

Since our specimens were not isotropic and homogeneous, the modulus measured was an effective modulus (in fact, the effective flexural modulus—see note in Section 1) and differences in the values obtained parallel and perpendicular, respectively, to the pressing direction were expected. In fact it turned out that the E values determined by means of the specimen flexural vibration with bending plane parallel to the pressing direction were lower than those determined perpendicular to the pressing axis (Fig. 2).

For evaluating the "experimental" moduli from measured resonant frequencies and geometrical dimensions of the sample, formulas (1) derived for homogeneous, isotropic and uniform samples made from a linearly

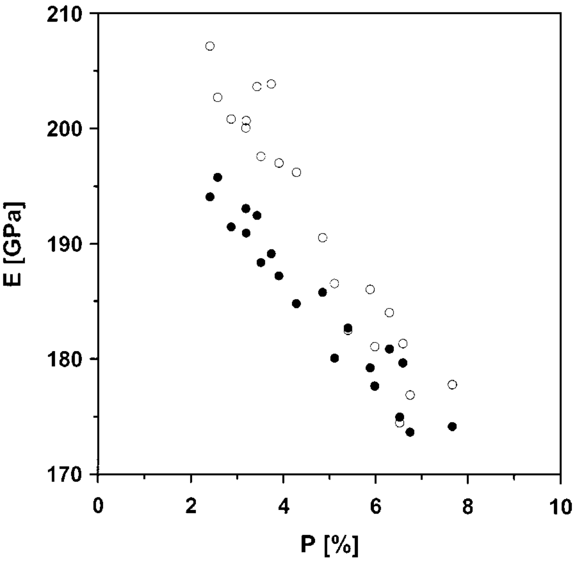

Figure 2 Young's modulus E as a function of a total porosity P. Values presented were determined in an experimental way from the specimen flexural vibration in the direction parallel (●) and perpendicular (○) to the pressing direction.

elastic material were used. If in this case variations in the moduli measured in various directions are observed, it must be assumed that the sample tested does not fulfill at least one of the given theoretical assumptions, i.e., it is not sufficiently linearly elastic, and/or not homogeneous and isotropic, etc. In the present case, of course, the latter mentioned fact, the layered structure of the specimen, is the reason for the deviation measured.

4. Vibration of quasilayered bars. Theory

As we noted in Section 1, Classical Lamination Theory (e.g [6]) provides different values of effective flexural moduli of actual layered materials when the bending occurs perpendicular and parallel to the layers.

To support the above statement in a theoretical way also for quasilayered bars, the standard "dynamical-resonant-method" evaluating procedure (Equation 1) was used for determining the Young's modulus values. But instead of experimentally measured resonant frequency, theoretically calculated natural frequency of a quasilayered bar was used as an "input parameter".

Somewhat simplified theoretical model of the real specimen was used for calculating the natural frequency. A rectangular bar of height H, width W and length L was considered. Properties of material of the bar (Young's modulus E, density ρ) were assumed as varying only in the direction of the bar height. Along the bar length and width the properties are constant. As the flexural vibration was used for experimental determining E on our samples, it was necessary to modify the Bernoulli-Euler beam theory (e.g. [8]) to be applicable to the quasilayered samples.

Vibration frequencies are obtained by solving the corresponding equation of motion [8]. The equation of motion is derived by means of the Hamilton's principle of minimal action. Lagrange's function, occurring in the expression for action, consists of the kinetic and potential (elastic) energies of a deformed bar. Required elastic energy is determined by means of the strain and stress tensor fields derived for a bent quasilayered bar under consideration. The geometry of deformation of material fibers and bar planar cross sections in a bent quasilayered bar is similar to the geometry of deformation in a homogeneous bar. Therefore, the strain tensor is qualitatively similar to the strain tensor in a homogeneous bar, that is, the relative fiber elongation (contraction) in a given point of the cross section increases linearly with increasing distance from the neutral axis of a given cross section. This neutral axis, in general, is perpendicular to the bending plane but does not pass through the centroid of the cross section, as it is in a homogeneous bar.

The stress tensor is determined on the basis of the Hooke's law by means of the above mentioned strain tensor and the nonhomogeneous distribution of the material Young's modulus values along the cross section.

The resultant elastic energy of a bent quasilayered bar, if expressed by means of the neutral-fiber curvature, differs from the elastic energy of a homogeneous bar only by a pre-factor; where the expressions $\iint_{\text{cross section}} E(h)h^2\,\mathrm{d}h\,\mathrm{d}w$ for bending in the HL plane or $\iint_{\text{cross section}} E(h)w^2\,\mathrm{d}h\,\mathrm{d}w$ for bending in the WL plane play the role of the flexural rigidity of the bar. Quantities h and w are distances of a given area element $\mathrm{d}h\,\mathrm{d}w$ from the cross section neutral axis for a given type of bending. Neutral-axis position within the cross section is determined by condition $\iint_{\text{cross section}} E(h)h\,\mathrm{d}h\,\mathrm{d}w = 0$ (bending in the HL plane) or $\iint_{\text{cross section}} E(h)w\,\mathrm{d}h\,\mathrm{d}w = 0$ (bending in the WL plane).

The kinetic energy of a quasilayered bar, if expressed by means of the velocity of the neutral-fiber transverse motion, differs from that of a homogeneous bar by the pre-factor $\iint_{\text{cross section}} \rho(h)\,\mathrm{d}h\,\mathrm{d}w$ instead of the $HW\rho_0$.

These differences lead to the analogous changes of corresponding quantities in the equation of motion and consequently in the relation for frequency of a homogeneous bar when they are rederived for the quasilayered bar.

Taking into account the conditions determining the neutral-axis position within the quasilayered bar cross section and transforming relevant expressions from the hw-frame to the co-ordinate frame determined by the cross-section edges (the origin at the vertex of the cross section, axis x oriented along the height of the bar) we get the following relations for the natural angular frequencies $\omega \equiv 2\pi f$ of flexural vibration of the quasilayered bar being considered:

$$\omega_{i\perp}^2 = \frac{n_i^4}{\rho H L^4}\left[\int_0^H E(x)x^2\,\mathrm{d}x - \frac{\left(\int_0^H E(x)x\,\mathrm{d}x\right)^2}{\int_0^H E(x)\,\mathrm{d}x}\right]$$

for vibration with the bending plane parallel to the HL plane (2a)

$$\omega_{i\parallel}^2 = \frac{n_i^4 W^2}{12\rho H L^4}\int_0^H E(x)\,\mathrm{d}x$$

for vibration with the bending plane parallel to the WL plane (2b)

Here ρ is the averaged bar mass density. n_i's are roots of the corresponding characteristic equation determined by the boundary conditions being applied (e.g., for a bar with free ends this equation is $\cos(n_i)\cosh(n_i) = 1$).

Substituting expressions (2a) and (2b) into the formula (1) used for evaluating the "experimental" moduli, and assuming that $L \gg W, H$, that is, $T = 1$ with great accuracy, the following relations for effective moduli are obtained:

$$E_{\perp}^{\mathrm{v}} = \frac{12}{H^3}\left[\int_0^H E(x)x^2\,\mathrm{d}x - \frac{\left(\int_0^H E(x)x\,\mathrm{d}x\right)^2}{\int_0^H E(x)\,\mathrm{d}x}\right]$$

for vibration in the *HL* plane, i.e., bending occurring perpendicular to quasilayers (3a)

$$E_{\parallel}^{\mathrm{v}} = \frac{1}{H}\int_0^H E(x)\,\mathrm{d}x \quad \text{for vibration in the } WL \text{ plane,}$$

i.e., bending occurring parallel to quasilayers (3b)

It should be noted that above expressions for effective moduli derived by means of natural frequencies of a quasilayered bar undergoing flexural-vibration test differ from those derived by means of an elongation of the same bar undergoing tensile test. Qualitatively similar results are provided by the Classical Lamination Theory for actual layered bars.

The expressions for "tensile" moduli can be derived by modifying the procedure used for evaluating the effective moduli for a slab model of composite materials (e.g. [6, 9]). For our quasilayered bar we obtain:

$$E_{\perp}^{\mathrm{t}} = \left(\frac{1}{H}\int_0^H \frac{\mathrm{d}x}{E(x)}\right)^{-1} \quad \text{for a tension perpendicular}$$

to quasilayers, i.e. along the direction of H (4a)

$$E_{\parallel}^{\mathrm{t}} = \frac{1}{H}\int_0^H E(x)\,\mathrm{d}x \quad \text{for a tension parallel to}$$

quasilayers, i.e. along the direction of L or W (4b)

Expressions for $E_{\parallel}^{\mathrm{v}}$ and $E_{\parallel}^{\mathrm{t}}$ are identical each other. For actual layered bars they obtain the form known as the Rule or Law of Mixtures, which one is often used for calculating the effective modulus of multiphase materials. Expressions for $E_{\perp}^{\mathrm{v}}$ and $E_{\perp}^{\mathrm{t}}$ differ from each other and from expressions for "parallel" effective moduli.

5. Some illustrative examples

Effective Young's moduli ($E_{\perp}^{\mathrm{v}}$ and $E_{\parallel}^{\mathrm{v}}$) were calculated for a number of bars with various distribution of material Young's modulus values along the bar height. Some of them are presented below to illustrate some characteristic features of these moduli.

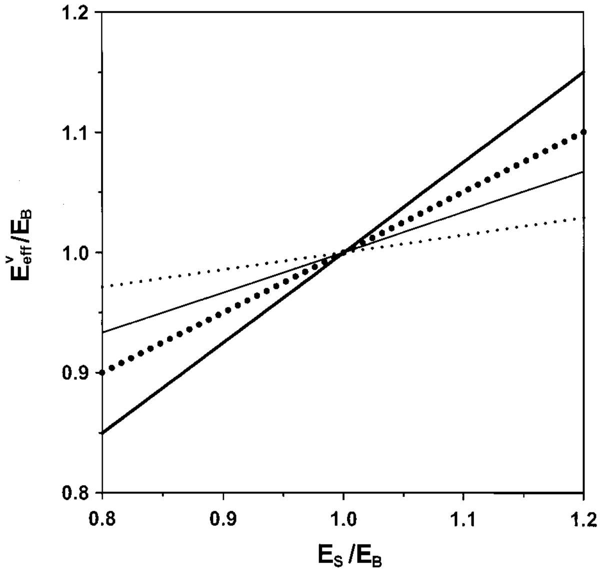

Figure 3 Effective Young's modulus $E_{\mathrm{eff}}^{\mathrm{v}}$ ($E_{\mathrm{eff}}^{\mathrm{v}} = E_{\perp}^{\mathrm{v}}, E_{\parallel}^{\mathrm{v}}$) as a function of the surface-to-bulk Young's moduli ratio $E_{\mathrm{S}}/E_{\mathrm{B}}$. Values presented were calculated in a theoretical way for a symmetric rectangular bar with the material modulus varying along the bar height linearly with the distance from the bar centre (thicker curves) and as the sixth power of the distance from the bar centre (thinner curves). Doted lines represent $E_{\parallel}^{\mathrm{v}}/E_{\mathrm{B}}$, solid lines $E_{\perp}^{\mathrm{v}}/E_{\mathrm{B}}$.

As an example of situations with continuously varying modulus values, the results for the case with symmetric ($E(\xi) = E(-\xi)$) modulus distribution characterized by a function

$$E(\xi) = E_{\mathrm{B}} + (E_{\mathrm{S}} - E_{\mathrm{B}})\xi^n \quad 0 \le \xi \le 1$$

are presented. Here ξ is a dimensionless coordinate determining a position along the bar height ($\xi = 0$ is for the centre of the bar, $\xi = \pm 1$ are for the surface). E_{S} is the surface value of the Young's modulus, E_{B} is that for the centre of the bar. Examples presented in Fig. 3 are curves calculated for $n = 1$ and for $n = 6$.

For all the cases investigated, $E_{\perp}^{\mathrm{v}}$ was lower than $E_{\parallel}^{\mathrm{v}}$ for $E_{\mathrm{S}} < E_{\mathrm{B}}$ and $E_{\perp}^{\mathrm{v}}$ was higher than $E_{\parallel}^{\mathrm{v}}$ for $E_{\mathrm{S}} > E_{\mathrm{B}}$. The differences between $E_{\perp}^{\mathrm{v}}$ and $E_{\parallel}^{\mathrm{v}}$, and between effective moduli and E_{B} decrease with increasing exponent n. It is due to the fact, that with increasing value of n the volume fraction of material with modulus equal or nearly equal to E_{B} increases.

The examples of effective Young's moduli for bars with discontinuous distribution of material modulus (actual layered bars) are shown in Fig. 4. The curves presented are the effective moduli calculated for a symmetric bar consisting of three layers. The material moduli within each layer are constant. Two outer layers are identical with respect to moduli values and geometrical dimension. For a such bar we have:

$$E_{\perp}^{\mathrm{v}} = E_{\mathrm{S}} + (E_{\mathrm{B}} - E_{\mathrm{S}})v_{\mathrm{B}}^3$$

$$E_{\parallel}^{\mathrm{v}} = E_{\parallel}^{\mathrm{t}} = E_{\mathrm{S}}v_{\mathrm{S}} + E_{\mathrm{B}}v_{\mathrm{B}} \quad \text{(Rule of Mixtures)}$$

$$E_{\perp}^{\mathrm{t}} = \frac{E_{\mathrm{S}}E_{\mathrm{B}}}{E_{\mathrm{S}}v_{\mathrm{B}} + E_{\mathrm{B}}v_{\mathrm{S}}}$$

Here E_{S} is the Young's modulus in outer layers, E_{B} in the central layer. v_{S} and v_{B} are volume fractions of the

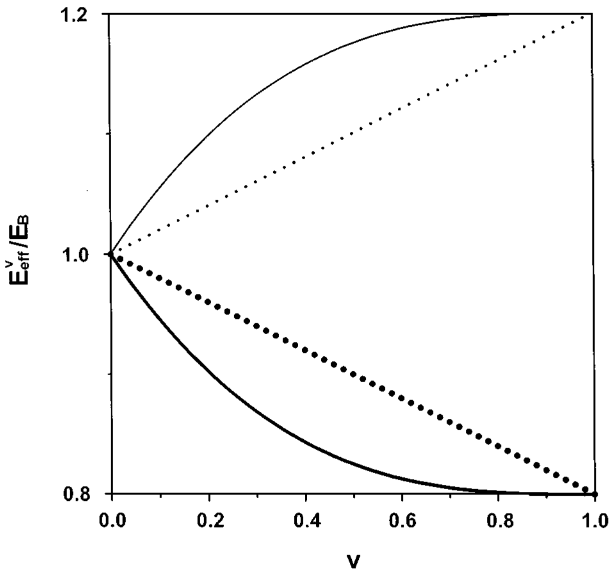

Figure 4 Effective Young's modulus $E^{\text{v}}_{\text{eff}}$ ($E^{\text{v}}_{\text{eff}} = E^{\text{v}}_{\perp}, E^{\text{v}}_{\parallel}$) as a function of the outer-layers volume fraction v. Values presented were calculated in a theoretical way for a symmetric rectangular bar consisting of three layers. The outer-to-central layer Young's moduli ratios $E_{\text{S}}/E_{\text{B}}$ were 0.8 (thicker curves) and 1.2 (thinner curves). Doted lines represent $E^{\text{v}}_{\parallel}/E_{\text{B}}$, solid lines $E^{\text{v}}_{\perp}/E_{\text{B}}$.

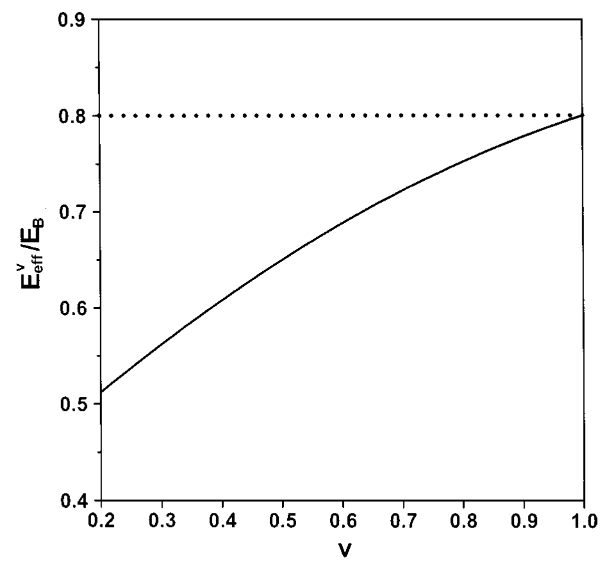

Figure 5 Effective Young's modulus $E^{\text{v}}_{\text{eff}}$ ($E^{\text{v}}_{\text{eff}} = E^{\text{v}}_{\perp}, E^{\text{v}}_{\parallel}$) as a function of the outer-layers volume fraction v. Values presented were calculated in a theoretical way for a symmetric rectangular bar consisting of three layers. Outer layers are porous with the Young's modulus linearly decreasing with the porosity, central layer is poreless with the Young's modulus E_{B}. The total porosity is assumed as constant and of amount that, being distributed homogeneously throughout the sample, it would lower the Young's modulus to the value $0.8E_{\text{B}}$. Doted line represents $E^{\text{v}}_{\parallel}/E_{\text{B}}$, solid line $E^{\text{v}}_{\perp}/E_{\text{B}}$.

outer layers and central layer, respectively. They obey the condition $v_{\text{S}} + v_{\text{B}} = 1$.

Also in this case $E^{\text{v}}_{\perp}$ is lower than $E^{\text{v}}_{\parallel}$ when the modulus of outer layers is lower than the modulus of a central layer, and $E^{\text{v}}_{\perp}$ is higher than $E^{\text{v}}_{\parallel}$ if the central-layer modulus is lower than that of outer layers.

Fig. 5 illustrates the statement that in the case of nonhomogeneous porosity distribution, macroscopic properties of specimens are determined not only by the total porosity but also by the distribution of porosity. In this case we investigated the symmetric bar consisting of three layers, too. Two outer layers are identical and porous with homogeneously distributed porosity. The central layer is poreless. The porosity causes the decrease of the effective Young's modulus of the outer layers. We assume that within the porosity range under consideration the Young's modulus decreases linearly with increasing porosity. We calculated the effective moduli $E^{\text{v}}_{\perp}$ and $E^{\text{v}}_{\parallel}$ for situations with a constant total porosity of the bar and with various outer layers heights. The requirement of constant total porosity leads to the change of the porosity within the outer layers when the height of these layers changed. Consequently, it leads to the change of the effective moduli of the outer layers with changing the volume fraction occupied by the outer layers within the bar.

Our calculations showed that within the model considered the effective modulus $E^{\text{v}}_{\parallel}$ depends only on the total porosity and it is independent on the porosity distribution for a wide class of distribution functions. On the other hand, the effective modulus $E^{\text{v}}_{\perp}$ is sensitive to the total porosity as well as to the porosity distribution (Fig. 5).

6. Conclusions

In this paper, the results of both experimental determinations and theoretical calculations of effective moduli of bars made from sintered iron, with a quasilayered porosity distribution caused by the process of production, are presented. These results can be recapitulated as follows:

(i) The samples used for experimental testing were quasilayered in the sense that porosity decreased from surfaces to the centre of bars only in one transverse direction (coincident with the direction of pressing during sample preparation). Along the other transverse direction as well as along the length of the bar the porosity was nearly constant. The effective Young's moduli derived from measurements of resonance frequency of the flexural fundamental mode with the bending plane perpendicular to quasilayers were lower than those derived by means of vibrations parallel to quasilayers (Fig. 2).

(ii) The general expressions (2a) and (2b) for resonance frequencies of flexural vibrations (and consequently for effective flexural moduli (3a) and (3b)—effective Young's moduli in terminology of this article) of prismatic quasilayered bars of rectangular cross section were derived in a theoretical way. In these expressions, the crucial role is played by the bar-material Young's modulus value as a function (continuous or step-like) of position along the bar cross section. For determining the position within the cross section, the natural coordinate frame connected with the cross-section edges is used. The expressions are closed in the sense that no preliminary determination of neutral-axis position is needed.

The following conclusions can be drawn from our theoretical investigations based on Eqs. (3a) and (3b):

(iii) For structural parts with a nonhomogeneous porosity distribution, the total porosity alone is insufficient for determining the macroscopic properties of these parts in some applications. Informations about the porosity distribution are also needed for predicting the behaviour of specimens in particular situations.

(iv) Effective Young's moduli evaluated by means of measured resonant frequencies of a rectangular quasi-layered bar undergoing flexural vibration of various polarization can really differ from each other. And in situations, when a material modulus of the surface regions is lower than the material modulus of the bulk of the sample, values of effective moduli provided by vibration perpendicular to layers are usually lower than the values provided by vibration parallel to layers. This theoretical result confirms qualitatively the initial assumption that the Young's modulus differences obtained in an experimental way are due to the "quasilayered" structure of the samples used. As the porosity in our samples was considerably concentrated near the surface (Fig. 1), the samples can be treated as layered bars with a lower modulus near the surface and a higher modulus in the core of the sample. And just the model systems with such distribution of material modulus values provide the same qualitative relations between $E_{\perp}^{v}$ and $E_{\parallel}^{v}$ as the experiments on our samples.

To compare our theoretical and experimental results also quantitatively, the detailed quantitative information on the porosity distribution throughout the bar cross section is needed. Determination of this porosity distribution represents the aims of current metallographic studies.

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References
1. R. HAYNES, "Mechanical Properties of Sintered Metals" (Freund, London, 1981).
2. R. CYTERMANN, *Powder Metall. Int.* **19** (1987) 27.
3. R. HAYNES and J. T. EGEDIEGE, *Powder Metall.* **32** (1989) 47.
4. J. R. MOON, *ibid.* **32** (1989) 132.
5. P. K. JONES, K. BUCKLEY-GOLDER, R. LAWCOCK and R. SHIVANATH, in Adv. Powder Metall. & Partic. Mater. 1996, Vol. 4, edited by T. M. Cadle and K. S. Narasimhan (MPIF, Princeton, 1996) p. 13.
6. R. F. GIBSON, "Principles of Composite Material Mechanics" (McGraw-Hill, New York, 1994).
7. S. SPINNER and W. E. TEFFT, *Proceedings, Am. Soc. Testing Mats.* **61** (1961) 1221.
8. R. R. CRAIG Jr., "Structural Dynamics—An Introduction to Computer Methods" (Wiley, New York, 1981).
9. T. H. COURTNEY, "Mechanical Behavior of Materials" (McGraw-Hill, New York, 1994).

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